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Summary Sheet

As the United States Southwest continues to face an unprecedented water crisis imposed by the significant impact of droughts on water reservoirs such as Lake Mead, the reclamation and reuse of wastewater is crucial for alleviating shortages in usable water. Wastewater recycling has become vitally important in establishing a permanent source of water for states during unprecedented times of drought. In this problem, we are tasked with coming up with a water reclamation plan for Lake Mead after rigorous research of Lake Mead's hydrological cycle, its patterns in water elevation, and the surrounding circumstances of the states which depend on Lake Mead as a source of water.

Before developing our plan, we first identified the relevant factors involved in the fluctuations of Lake Mead's long term water levels. Firstly, we **verified the methodology** involved in the calculation of Lake Mead's relationships in water elevation, surface area, and volume. By determining water elevations using on field bathymetric data, surface area can be determined using an application of the Riemann integral, and volume relationships can be determined through the sum of consecutive fustrums inherent in the lake's geometry. These confirmations would lead to **an analysis of the inflow, outflow, and loss factors for Lake Mead's hydrological cycle**, and through rigorous research, we identify common factors in Lake Mead's water fluctuations and condensed these parameters in a comprehensive equation for changes in water capacity for the reservoir.

In our model development, we used the previously created comprehensive equation for Lake Mead's water capacity to develop a water elevation which to model Mead's future water levels using recently acquired water elevation data. This is accomplished by **an exploration of the available data** where we observed that recent water elevation patterns showed a tendency for high intensity droughts with low variability. Thus, we employed a sinusoidal model on the residuals formed by the linear regression model of a transformed representation of Lake Mead's monthly changes in water elevations to predict annual water elevation levels. Subsequently, we applied our model to water elevation levels during both the recent drought period and water elevation data for the last fifteen years to predict future annual water levels in 2025, 2030, and 2050. Our models concluded that **Lake Mead would face startling losses of water** regardless of previous circumstance. Thus, we identified and described key factors in water reclamation initiatives to prepare for a comprehensive water reclamation plan that can utilize current wastewater recycling technology to alleviate the current water crisis.

We propose a federal, large-scale water reclamation project based on a two-pronged approach of direct federal **construction of water treatment facilities** through federal and local resources and outsourcing funds to pre-established agencies and local companies in the construction of practical water treatment technology. Finally, through long-term analysis using predetermined statistical measures, the impact of our water reclamation plan can be measured.

Keywords: Hydrological Cycle, Bathymetric Data, Riemann Integral, Inflow, Outflow, Loss, Linear Regression, Sinusoidal Model, Water Treatment Facilities

CONSERVATION

A RECYCLING PLAN TO SAVE LAKE MEAD

BEFORE IT DRYS UP

The Recent Trend in Lake Mead

Through extensive modelling and analysis of Lake Mead's water cycle, we determined that Lake Mead's water levels are dropping quickly as droughts become more intense. By 2050, our models predict that Lake Mead could lose up to 200 feet in water elevation — a drop that could spark a water crisis in the American Southwest.

Our Goal in Water Reclamation

The dramatic decrease in water levels shown by our modelling presents a water crisis we must solve now. With water reclamation, we hope to establish independence from Lake Mead as a source of water and to generate the opportunity of artificially augments Mead's water levels. Water reclamation will be new major source of water for the American Southwest.

OUR VISION

COLLECT DATA ANALYTICS

CONSTRUCT TREATMENT FACILITIES

WORK WITH THE COMMUNITY

PUBLIC OUTREACH

IMPACT ANALYSIS

SAVE OUR WATER

PLANNING

The wastewater reclamation project will affect the three major US beneficiaries of Lake Mead's water: Arizona, Nevada, and California. Through a two pronged approach of federal and local intervention, the plan calls for the construction of large-scale water treatment facilities and the construction of a vast array of water infrastructure to transport treated wastewater.

- The plan hopes to accommodate all water quality needs from the municipal, industrial, and commercials sectors as community feedback is used to improve the reclamation process.
- Leftover treated water will be use to augment Lake Mead's water levels as these states become independent from the reservoir.
- Throughout the project, statistical analysis will be applied to all facets of the program to create a detailed perspective on water reclamation.

Contents

1 Lake Mead Water Dynamics

1.1 Question Restatement

The general purpose of this section was to construct an equation that gives us an effective and reliable representation of lake water dynamics through the analysis of changes in lake water storage.

- First, we should focus on the veracity of the elevation, area, and volume relationships given by the Bureau of Reclamation in 2010. In the following work, we showcase the information and methodology required to verify the data provided.
- Second, we should identify and explain factors of inflow, outflow, and loss in Lake Mead. We then need to create an equation that models the annual change in lake storage from selected factors involved in Mead's current capacity

1.2 Assumptions

- **Assumption 1 :** The geometric shape of a lake can be defined as a series of unique frustums stacked against each other.

  Justification 1 : The nature of the data collection from a bathymetric survey leads to the creation of conical levels within a lake separated by determinant elevations. Thus, these levels create frustums with irregularities accounting for itself due to the stochastic nature of lake generation creating structures that both increase and reduce the final capacity of a lake.

- **Assumption 2 :** Contours from subsequent elevations in Lake Mead should carry a consistently similar geometric shape in terms of surface area.

  Justification 2 : The formation of Lake Mead was from a vertical perspective where lower elevations carried smaller contours, Thus, the gradual deepening of the Lake would lead to a similarly gradual reduction of the lake's maximum contour. Thus, the contours of Lake Mead follow a similar shape throughout its depth.

- **Assumption 3 :** The combination of echo-sounding and GPS tracking provides accurate positional and depth-wise information on Lake Mead

  Justification 3 : A standard in the industry, echo-sounding and GPS tracking is used by the US Geological Survey for its bathymetric surveys. Thus, the elevation data determined by such methodology could be considered accurate.

- **Assumption 4 :** A discussion about the volume of Lake Mead about the relationships between hydrological factors can be applied to a discussion about the water level of Lake Mead with little change.

  Justification 4 : Both the volume and water level of Lake Mead refer to the basic core of the reservoir's current capacity. Thus, discussions about the two is interchangeable with the only notable difference between the two being the unit discussed.

1.3 The Importance of Bathymetric Data

Bathymetric data is information about the depths and shapes of underwater terrain. Bathymetric data is obtained through the combined usage of a GPS receiver and a multi-beam depth sounder. The multi-beam depth sounder uses a method of underwater mapping called echo sounding where sound pulses are emitted from below a survey ship, reflects off the ocean bottom, and then returns to the surface where it is detected [1, 2].

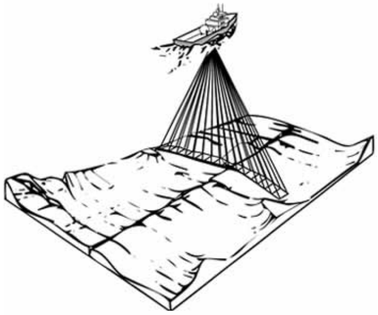

Figure 1: The geometry of echo sounding for mapping ocean bottom contours. [3]

The depth is then calculated using the speed of sound in water and measuring the time it takes the sound to travel to the bottom and return. While the multi-beam depth sounder collects depth information, a GPS receiver collects the ship's positional arguments, and together, the data creates a detailed survey of a water reservoir's positional data. Using the bathymetric data for Lake Mead, the reservoir's irregular shape and varying depths can be captured, and the relationship between elevation, area, and volume of Lake Mead can be determined with this positional information alone.

1.4 Confirming Surface Area Data.

The bathymetric data obtained from 1.3 could be superimposed over an outline of the lake created from an aerial photo or satellite image. The result is a bathymetric map which visualizes the bathymetric data along with a shape of Lake Mead [2].

From the bathymetric map, various methods are possible to determine the surface area of Lake Mead for each elevation measured by the Bureau of Reclamation. One such method to determine the surface area involves applying the underlying principles behind 2-dimensional Riemann sums:

$$\sum_{A}\sum \sqrt{\left[\frac{\partial f(x,y)}{\partial x}\right]^2 + \left[\frac{\partial f(x,y)}{\partial y}\right]^2 + 1}\Delta A \tag{1}$$

Where $f(x, y)$ describes the "height" of the surface at a coordinate, the surface A is divided into small "cells" (area elements) ΔA [and we're summing those small cells over the whole region, multiplied by a "scale factor"], and the "scale factor" describes the way that the area changes over a small distance.

Although the formula itself may be complex, the underlying principle is merely to divide a region into smaller, simpler "cells", and then to use those (multiplied by the appropriate

Figure 2: A simple bathymetric map of a reference lake. [4]

scale factor) to calculate the final surface area. In the case of the irregular shape of a lake, the individual pixels of a high-quality bathymetric map act as ”cells” to calculate the surface area of any elevation within Lake Mead. By calculating the number of pixels under a given elevation's area in Lake Mead, and multiplying it by the image's appropriate scale factor, the surface area can be determined. Indeed, because the surface of the lake is more or less flat, the ”scale factor” reduces to a constant, dependant on merely the scale of the map.

1.5 Confirming Volume Data

Using the surface-area calculations in **1.4**, and the acquired elevation data from bathymetric analysis, Lake Mead's volume can be calculated from the following basic equation: [5]

$$V = \frac{1}{3}H\left(A_0 + A_1 + \sqrt{A_0 \times A_1}\right) \tag{2}$$

Where:

- V= the volume of the water;
- H = the difference in depth between 2 successive depth contours:
- A_0= the area of the lake within the *outer* depth contour being considered;
- A_1= the area of the lake within the *inner* depth contour being considered

The equation rests on the two assumptions that a lake's geometric shape can be defined as a series of frustums where the contours are of similar shape (Assumption 1, 2). Thus, through the above equation, the volume between two successive elevations of Lake Mead could be calculated as the volume of a frustum, and any elevation's volume could be calculated through the summation of it's successive depths. For contour surface areas from A_0 to A_k:

$$V_{total} = \sum_{i=1}^{k} \frac{1}{3} H \left(A_i + A_{i-1} + \sqrt{A_i \times A_{i-1}} \right) \tag{3}$$

Where the volume of A_0 to A_k where the final A is the desired elevation's surface area and A_0 is the surface area at the bottom of the lake.

1.6 Factors of Inflow

There are several characteristics that are indicative for the amount of water entering Lake Mead as inflow:

- **Direct inflow from tributaries**: Lake Mead receives water from four tributaries: the Colorado River, the Las Vegas Wash, and the Virgin and Muddy Rivers. These tributaries are also affected by factors of inflow, outflow, and loss. [6]
- **Precipitation**: The portion of the hydrological cycle in which atmospheric water vapor is condensed, falling to earth. Along with direct inflow from tributaries, precipitation is a key factor of inflow as other factors of inflow rely on precipitation. [7]
- **Runoff**: A direct consequence of precipitation, leftover water is pulled by gravity across the surface of land into river's, streams, or lakes. [8]
- **Groundwater Recharge**: As water from precipitation experiences runoff, some water seeps into the earth. Eventually, the water could be reintroduced into the hydrological cycle by moving through pores down to a water basin. [9]
- **Drainage**: Similar to runoff, water is pulled by gravity across the surface of land into water basins. However, the origin of this water arrives from melted snow or ice above the beneficiary water basin. [10]

1.7 Factors of Outflow

There are several characteristics that are indicative for the amount of water leaving Lake Mead as outflow:

- **Supplying the American Southwest and Mexico**: As a closed water reservoir, Lake Mead acts as an important source of water for multiple states in the American Southwest including Nevada, Arizona, and California, and the reservoir also serves as a source of water for Mexico. [11]

1.8 Factors of Loss

There are several characteristics that are indicative for the amount of water lost in the hydrological cycle of Lake Mead:

- **Evaporation**: After sufficient heat, water at the surface turns into water vapor, and evaporation rates have greatly increased as a result of climate change induced droughts. [12]
- **Infiltration**: As water undergoes runoff, some water droplets infiltrate into subsurface soil and rock where it will gradually move vertical and horizontally into the water reservoir to be discharged. [13]
- **Depression Storage** : As groundwater undergoes infiltration, some water is lost because it becomes trapped in small depressions within the ground, preventing the possibility for escape into a water reservoir.[14]

1.9 Comprehensive Model On Water Dynamics

Using Guinaldo et al.'s model [15] as a framework, the equation of the annual change in the lake storage of Lake Mead as a function of time can be calculated as such:

$$\frac{dV_{\text{lake}}}{dt} = P_{\text{ol}} - E_{\text{ol}} + R + D + Q_{\text{in}} - Q_{\text{out}} - Q_{\text{gwd}} + Q_{\text{gwr}} \tag{4}$$

Where:

Symbol	Definition
$\frac{dV_{\text{lake}}}{dt}$	The change in volume of Lake Mead with respect to time.
P_{ol}	The volume of water Lake Mead gains as a result of precipitation over the lake.
R	The volume of water flowing into Lake Mead from runoff.
D	The volume of water flowing into Lake Mead from drainage.
Q_{in}	The volume of water flowing into Lake Mead from its four tributaries.
Q_{out}	The volume of water flowing out of Lake Mead due to use by either Nevada, Arizona, California, or Mexico.
Q_{gwd}	The volume of water withheld from Lake Mead as a result of various sources of ground water discharge such as infiltration and depression storage.
Q_{gwr}	The volume of water flowing into Lake Mead as a result of ground water recharge

From our equation on the annual annual change of the amount of water stored in Lake Mead, several relationships between parameters involved in Lake Mead's water capacity become apparent:

- Outflow for Lake Mead is dependent on solely intake from the American Southwest and Mexico as the artificial lake's purpose is to act as a water reservoir for municipal, agricultural, and industrial use.

- Many factors of inflow such as runoff, drainage, and groundwater recharge are based upon precipitation, and many factors of loss such as infiltration and depression are based upon evaporation.
- The importance of evaporation and precipitation for inflow, outflow, and loss leads to the conclusion that important factors for evaporation and precipitation become naturally important for changes in water elevation. Thus, the impact of droughts and wet seasons are crucial in Lake Mead's hydrological cycle.

2 Modelling of Lake Mead's Water Levels

2.1 Question Restatement

The general purpose of this section was to construct a model that gives an effective and reliable estimate of Lake Mead's future water levels.

- First, we should analyze the data provided about Lake Mead water levels on a monthly and annual high-low basis. In the following work, we highlight important characteristics of the data that are crucial for the later modelling.
- Second, we should define a set of criteria for drought periods and identify the beginnings and ends of periods of drought. Then, using this information, we then need to analyze and discern notable differences between the current drought period and earlier periods.
- Finally, we need to develop two models that predict Lake Mead's water level in the year 2025, 2030, and 2050; one model will use data from only the most recent drought period and the another model will use water level data from 2005 - 2020.

2.2 Assumptions

- **Assumption 1 :** The hydrological cycle can be defined as a series of dry seasons and wet seasons.

  Justification 1 : As a natural consequence of the hydrological cycle, the process of evaporation increases the capacity of water available for precipitation which deposits this capacity of water to undergo evaporation. This cyclical process of evaporation and precipitation leads to the broader hydrological cycle being defined by dry season followed by wet seasons.

- **Assumption 2 :** The monthly water elevations for the years 1952-1959 are model representations of a baseline hydrological cycle of a dry season and a wet season.

  Justification 2 : A model representation of a baseline hydrological cycle of a dry season and wet season should be a stark decrease in water levels followed by a stark increase in water levels in a 5-10 year period. The duration is meant to prevent the inclusion of any unusual conditions of hydrological cycles due to abnormal duration.

- **Assumption 3 :** Lake Mead has a baseline change in monthly water level where any change below it signifies a dry season and any change above it signifies a wet season.

  Justification 3 : Following Assumption 1, the lack of any middle ground between the wet season and the dry season would signify a level of change in monthly water level that determines the strict binary designation of a wet season or a dry season.

- **Assumption 4 :** The mentions of droughts in this analysis will only focus on hydrological droughts as a form of drought. [16]

  Justification 4 : The four major definitions of droughts are meteorological, agricultural, hydrological, and socioeconomic. The only droughts that could be adequately described with water elevation data are hydrological droughts. Thus, any mention of the term drought would be referring to hydrological droughts.

2.3 Exploratory Data Analysis of Lake Mead's Water Levels

The provided data on Lake Mead comes in two parts:

- The Low-High Elevation dataset provides the annual lowest and highest water elevations at Lake Mead from 1935 - 2020. Additional information includes specific date and time of data collection.
- The Monthly Elevation dataset provides the elevation of water at Lake Mead at the end of each month from January 1935 to September 2021.

We decided to analyze and compare the general trend of monthly water elevation, annual low water elevation, and annual high water elevation. However, a summarial purview of the data's line plot leads to our inference that the general trend of annual high water elevation would suffice due to the data's similarity in shape.

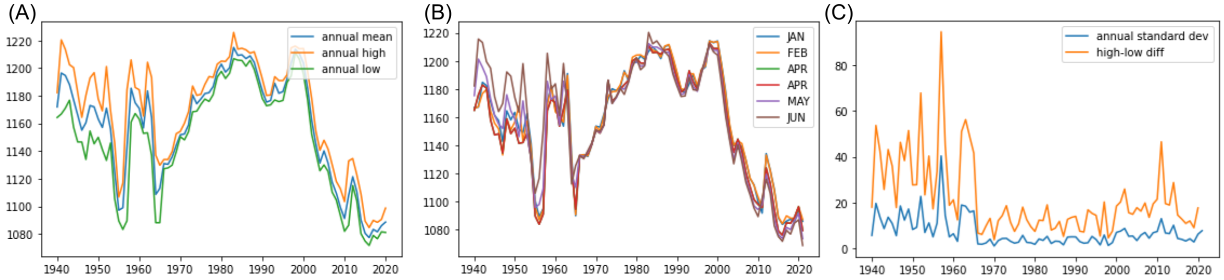

Figure 3: (A) Annual high, annual low, annual mean (B) All months collected with annual mean (C) Annual standard deviation vs difference between year high and low

Thus, we chose to focus the brunt of our analysis on the Monthly Elevation dataset, as the dataset contains a larger amount of elevation data — an important factor in generating a precise model. Furthermore, the patterns within the difference between the annual high water elevation and the annual water low elevation could also be mirrored within the annual standard deviation of monthly water levels. This similarity is shown in Figure 3. Conclusions we make with the annual standard deviation of monthly water levels could be cross applied to conclusions with the annual difference between the highest and lowest water elevations.

Within the annual standard deviation of monthly water levels, a Mann-Kendall Trend Test reveals an alternate hypothesis of a decreasing trend in the data with the standard deviation of annual water levels decreasing as time passes (p= 0.0163). As time progresses annually, monthly water elevations in a given year seem to concentrate themselves to a greater extent.

A Mann-Kendall Trend test for the monthly water levels for Lake Mead reveals similar results with an alternate hypothesis of a decreasing trend in the data (p =0.0338) . The decreasing trend in water levels also leads to the conclusion that either there exists a higher proportion of droughts as time increases or the severity of droughts has increased over time. As droughts become either more severe or more prominent, water levels decrease as wet seasons lose statistical significance.

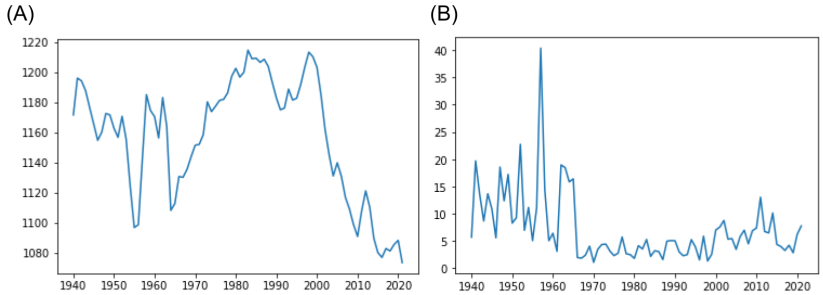

Figure 4: (A) Downward trend annual mean water elevations (B) Downward trend of annual standard deviations of monthly water levels

From the Exploratory Data Analysis we can conclude a number of observations:

- The patterns shown in the monthly elevation of water levels for Lake Mead can be cross-applied to annual low-high water elevations for Lake Mead.
- As time progresses, the range of possible water levels for a given year shrinks as the standard deviation of annual water levels decreases.
- The monthly water levels for Lake Mead shows no apparent trend for either an upward or downward trend in the dataset.

2.4 Drought Criteria

As defined by the US National Oceanic and Atmospheric association, a drought is defined as “a deficiency of precipitation over an extended period of time (usually a season or more), resulting in a water shortage.” We simplify this definition to have droughts be an extended period of time where water level growth is subpar (Assumption 4).

Our criteria for a drought is based on the previously made assumption that the hydrological cycle is made of dry-seasons and wet-seasons (Assumption 1). Thus, the lack of a middle ground between the two seasons means that we can assert a baseline change in water elevation for Lake Mead (Assumption 3). Any monthly changes in water level below this baseline would classify the corresponding month as a dry season, and any monthly changes in water level above this baseline would classify the corresponding month as a wet season. This binary classification helps create a database of dry and wet seasons to be later classified into droughts.

We identified the baseline change in water elevation by taking the 50th percentile change in water elevation for our framework hydrological cycle of the years 1952 to 1959 (Assumption 2). Then, our binary classification of dry-seasons and wet-seasons was used to filter out months with a monthly change in water elevation that was below the baseline.

However, this definition of baseline change in water elevation could result in short, anomalous decreases in water elevation being defined as drought. To combat this, we define a change in water elevation as the percentage change between the target month's water elevation and the water elevation of the month one year prior. This new definition of water elevation change eliminates the stochastic existence of outlier water elevations, and though this definition does include weak droughts with minimal severity, our definition of a drought seeks to define possible dry seasons that, over an extended period, would lead to a water shortage. The existence of minimal droughts only represent cases where wet seasons were closely available to prevent a large water shortage. Furthermore, to remove the existence of long dry seasons with little impact on water elevations, we set the minimum length of a drought to be 9 months based on historical conditions of the length of a dry season and the climatic conditions of Lake Mead.

The resulting complete definition of a drought is as follows:

- A drought consists of a period of dry-seasons which are defined as months with a change in monthly water level that falls below a predetermined baseline.
- Change in monthly water elevation is defined as the percentage change between the target month's water elevation and the water elevation of the month one year prior.
- The period of dry season which makes up a drought must last longer than 9 months.
- The beginning of a drought would be classified as the inciting month that reaches below the baseline change in water elevation.
- The end of a drought would be classified as the inciting month that reaches above the baseline change in water elevation.

2.5 Analysis of the Most Recent Drought Period

From our complete definition of a drought, the most recent drought period could be defined as the period of time between May 2020 and September 2021. Due to its status as a current drought and its correlation with Lake Mead's current water level crisis, this drought period was analyzed with close scrutiny to evaluate its differentiating qualities compared to previous drought periods.

2.5.1 Previous Wet Seasons and Droughts

Preceding the current drought was a series of wet seasons and droughts which contained a heavy bias towards droughts. This disparity can be seen in the Mann-Kendall Trend test performed in the data analysis earlier which shows a downward trend of water elevation caused by more droughts and less wet seasons. The current drought of sixteen months continues this pattern.

2.5.2 Inciting Elevation

The inciting elevation of the current drought period is unusually low with an inciting drought elevation of 1091.32 feet above sea level. The inciting elevation is 1.73 standard deviations below the 50th percentile which makes it the bottom 4% of inciting drought elevations. The low inciting elevation continues to illustrate the lack of wet seasons before the current drought period.

2.6 The Water Elevation Model for the Most Recent Drought Period

In order to achieve a summarial understanding of the mathematical patterns hidden within the water elevations of the recent drought period, we used a naive linear regression model to understand the possible model's greater complexities.

A preliminary linear regression model of monthly water elevation with respect to time does yield a model with some resemblance of following Lake Mead's monthly water elevations.

$$\underbrace{y_i}_{\text{dependent variable}} = \beta_0 + \beta_1 \cdot \overbrace{x_i}^{\text{independent variable}} \tag{5}$$

However, analysis of the residuals led us to conclude that the current representation of Lake Mead's water elevation was unsuitable for modeling. The monthly water elevation carries stochastic elements that makes modeling either inaccurate or impossibly complex. To resolve this issue, we transformed the data into an iterative function which calculates a running net sum of the percent changes in monthly water level between the current month and the previous year:

$$N(t) = \sum_{i=1}^{t} C(i) \tag{6}$$

Where $C(t) = \frac{M(t)}{M(t-12)-1}$ and $C(1)$ to $C(12)$ is 0 to prevent errors in calculating the percent change between two months.

An alternative equation for N(t) is a recursive definition of:

$$N(t) = N(t-1) + C(t) \tag{7}$$

with $N(t-1)$ being the net sum of percentage changes in monthly water levels for the previous month.

This definition of net change in monthly water level accounts for the stochastic nature of the dataset; the result is a more discriminate series of data which facilitates modelling.

A preliminary linear regression model for this recursive dataset shows significant signs of a sinusoidal model for the residuals between the fitted linear regression model and the genuine recursive data.

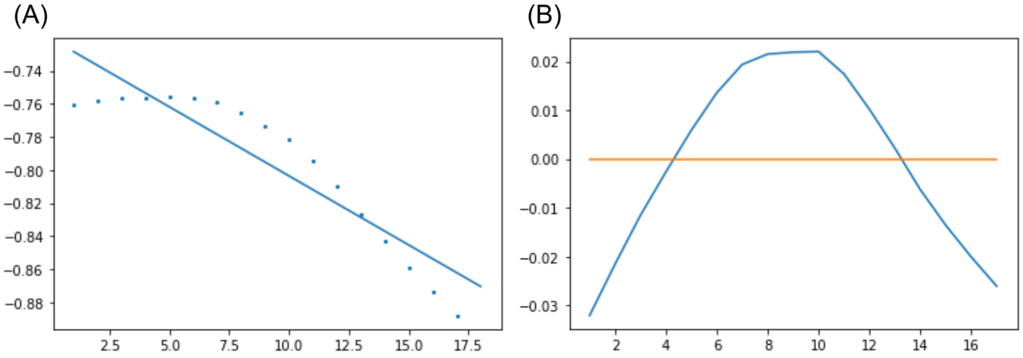

Figure 5: (A) Comparison between linear regression predictions and transformed dataset (B) Residuals plot of linear regression predictions and transformed dataset

Combining the linear regression model with the sinusoidal model leads to a final model which predicts the net percent change in monthly water levels:

$$N(t) = (B_0 + B_1 t) + B_1 \cdot B_1^t \cdot \sin\left(B_3 \cdot (t + B_4)\right) + B_5 \tag{8}$$

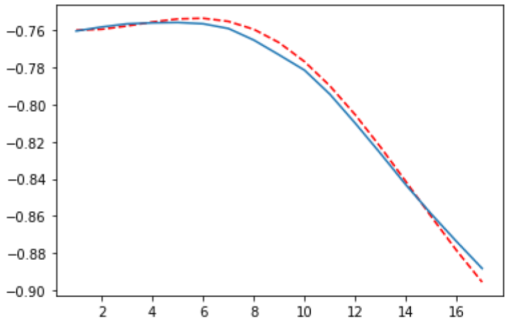

Figure 6: Comparison of combined model's predictions with net water change

In order to get the monthly percent changes in water levels, we can use our alternative definition of N(t) to create the following equation:

$$N(t) - N(t-1) = C(t) \tag{9}$$

Keeping in mind the definition of $C(t)$, we can find the monthly water level in one year for any month t months away from May 2020 using the following regressive model:

$$(C(t) + 1)(M(t)) = M(t + 12) \tag{10}$$

To convert the monthly water levels to an annual prediction, we used our model to determine the water levels for all months in the target year, and then, we calculated the mean of the water levels to achieve an annual prediction of water elevation in Lake Mead, where t is *years* from 2021 and t must be greater than or equal to zero:

$$Y(t) = \frac{1}{12} \sum_{i=7+12t}^{7+12t+11} M(i) \tag{11}$$

2.6.1 Making Predictions

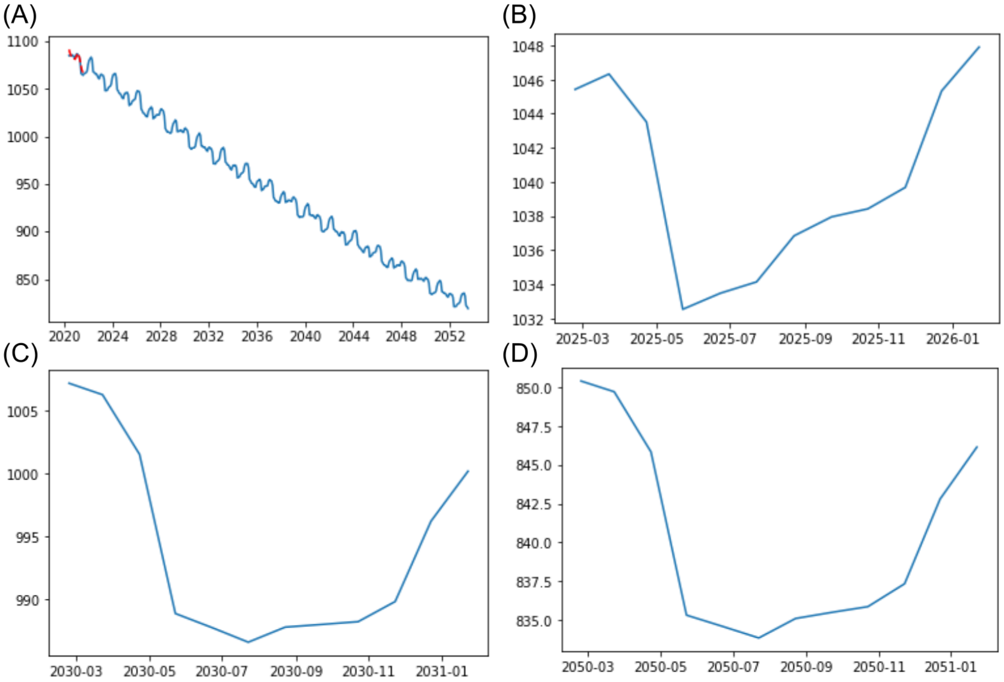

Figure 7: (A) All predictions from 2020 through 2054 using data from the recent drought period (the red line indicates available data) (B) The 2025 plot, the prediction is 1040.122 feet (C) The 2030 plot, the prediction is 994.032 feet (D) The 2050 plot, the prediction is 840.207 feet.

2.7 The Water Elevation Model for the 2005-2020 period

A similar methodology was applied to water levels from 2005-2020. We applied a linear regression model to the net cumulative water level change dataset and identified a sinusoidal pattern within the residuals. After acquiring the sinusoidal model, we converted the net-change into monthly water levels to be averaged and transformed into an annual water level prediction model.

2.7.1 Making Predictions

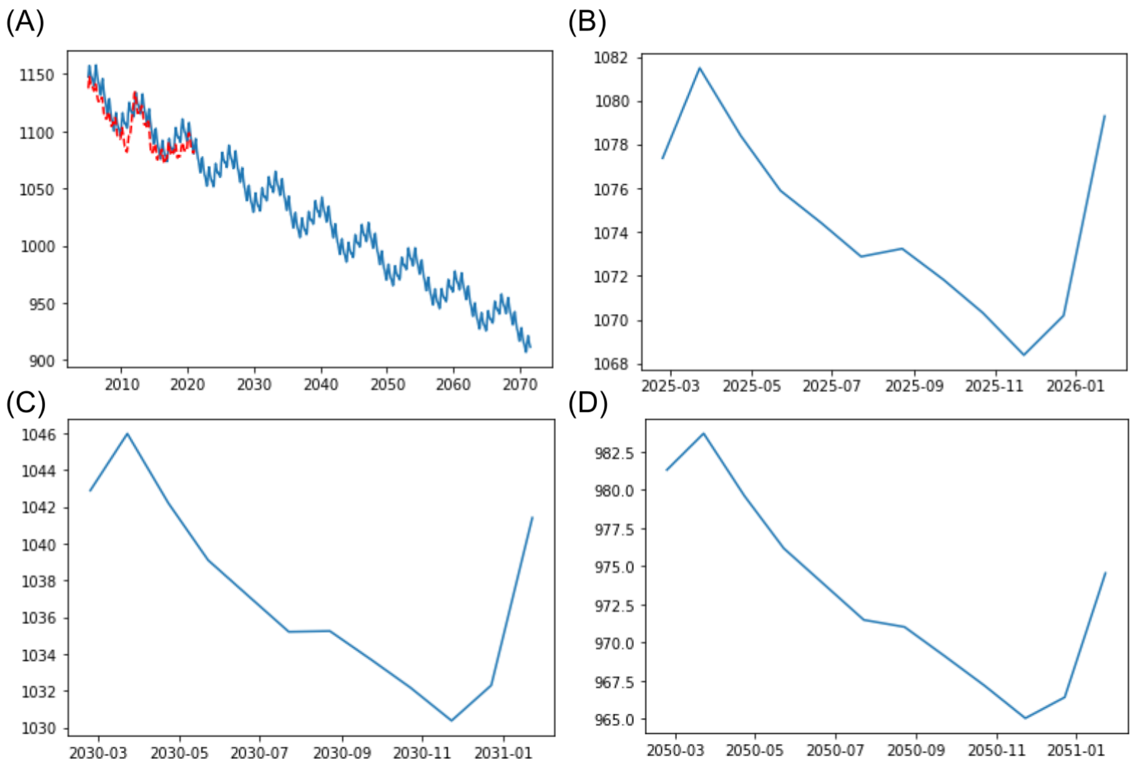

Figure 8: (A) All predictions from 2005 through 2072 (the red line indicates available data) (B) The 2025 plot, the prediction is 1074.482 feet (C) The 2030 plot, the prediction is 1037.302 feet (D) The 2050 plot, the prediction is 973.278 feet.

2.8 Strengths of Model

Our model is easy to implement and understand. The general architecture of a sinusoidal representation of residuals from a naive linear regression model can be applied to a large majority of timeframes. Regardless of the existence of wet seasons in the dataset, the model is still able to competently assess future water elevations by treating wet seasons and droughts as cyclical occurrences.

Due to the breadth of data fed to our model by adapting monthly water elevations to an annual prediction, the model is able to accurately predict variation in the data as the model accounts for monthly maximums and minimums that may impact final monthly water elevations. This makes our model very **robust** against interyear water level variation.

Furthermore, when the model acquires a complete cycle of droughts and wet seasons within its dataset, the model is able to accurately calculate the timings of droughts and wet seasons. This makes the model capable of predicting future water elevations with little deviation from the actual water level.

2.9 Weaknesses of Model

Our model tends to overestimate the loss in water elevations for future timeframes if it's given an incomplete picture of the lake's hydrological cycle. This lack of information can arise from the dataset only capturing a drought or a wet season, and thus, the model assumes that the lake can only be in a wet season or a drought. Due to this issue, the model does need a complete and comprehensive dataset which details a complete hydrological cycle of a drought and wet seasons.

Our model tends to require a large amount of data to accurately represent future water levels as it requires the monthly water levels of a completely hydrological cycle of a drought and wet season within a year before the target cycle to accurately predict the change parameter needed to fit the model. The model requires substantive water elevation data across a maximum of multiple years to achieve an accurate, long-term analysis of the lake's water levels.

3 The Wastewater Recycling Plan

3.1 Question Restatement

The general purpose of this section was to create a feasible plan for wastewater recycling to alleviate the burden held by water reservoirs like Lake Mead through an alternative methodology in obtaining clean water.

- First, we should identify and describe important factors to consider in a plan to recycle wastewater. These factors should address the multiple complexities within a plan for water-recycling such as important financial and logistical points of interest.
- Second, we should create a comprehensive federal plan for recycling wastewater in states heavily affected by droughts. We then need to create statistical measures which record the impact of implementing our plan.

3.2 Assumptions

- **Assumption 1 :** A wastewater recycling plan for Lake Mead must be implemented for all states which actively use Lake Mead.
- **Justification 1 :** Lake Mead acts as a water reservoir for several states, and any wastewater recycling plan which disregards a state using Lake Mead would hamper impact recognition and contradict the primary purpose of alleviating the burdens of Lake Mead.
- **Assumption 2 :** A wastewater recycling plan for Lake Mead does not need to be implemented for Mexico.
- **Justification 2 :** As a country which uses Lake Mead as a reservoir, the case of Mexico is difficult to account for due to the added complexity of international politics.
- **Assumption 3 :** As long as the proposed wastewater recycling plan is feasible and has the probability of creating clean water, the plan will pass through lawmaking barriers that may hinder its feasibility.

- **Justification 3 :** The willingness of US citizens to accept water conservation as a key concern and the potential costs of importing water means that lawmakers would not be concerned about the idea of a wastewater recycling plan as long as the plan appears feasible and impactful. However, if the plan appears lackluster during application, the plan's reduction or removal may be a concern to review. [17]

3.3 Important Factors to Consider in a Wastewater Recycling Plan

The factors to be considered in a wastewater recycling plan can be broken into three categories: treatment design, resource management, and public confidence. Factors related to treatment design deal with issues directly responsible for the design of the wastewater recycling plan. Treatment design factors are determined before practical application and act as the foundation behind the wastewater recycling plan. Resource management factors are factors dealing with the allocation and management of limited resources needed for the plan. Limited assets such as energy, manpower, and chemical resources are important factors to consider within a wastewater recycling plan. Finally, public confidence factors pertain to any factor which deals with public perception of the plan; if public confidence is stifled for a wastewater recycling plan, the resulting backlash can negatively influence the implementation for our plan.

3.3.1 Treatment Design

There are several important factors pertaining to treatment design in a water recycling plan for Lake Mead.

- **Type of Water Reuse Application:** Depending on treatment needs, the type of water treatment application may differ. For large operations, Effluent Treatment Plants are useful for cleaning large amounts of industrial wastewater, and for small businesses, common and combined effluent treatment plants are more affordable methods for smaller companies and operations. [18]
- **Reclaimed Water Quality Objectives:** The objectives of the reclaimed water are important to consider in designing a wastewater recycling program. Whether the program focuses on agricultural, municipal, or industrial use is vital for allocation of resources in treating wastewater. [19]
- **Compatibility with Existing Conditions:** A water treatment program should be harmonious with pre-existing government structures and inherent socioeconomic characteristics of target regions. Government agencies with similar goals as our water recycling plan should assist in the program, and large water treatment facilities should use permits for unused, available land.
- **Processing Flexibility:** The water reclamation plan should be flexible in its processing capabilities to prevent large scale removal and reconstruction. Flexible design such as diversity in water treatment facility types makes the program robust against future modifications.

3.3.2 Resource Management

There are several important factors pertaining to resource management in a water recycling plan for Lake Mead.

- **Operating and Maintenance Requirements:** Applications in recycled water required active maintenance and operation. Factors such as the personnel required, the degree of human intervention needed for water treatment, and the necessary resources for maintaining supply of clean water are important to consider for water reclamation.
- **Energy and Chemical Requirements:** Water treatment facilities need to account of energy and chemical requirements, and any plan surrounding water recycling must account for the costs of energy and chemical consumption, logistics of energy and chemical requirements, and efficiency in energy and chemical usage. [20]
- **Personnel and Staffing Requirements:** Aside from the personnel and staffing requirements for operating water treatment, personnel for other facets of the water recycling plan such as communicating with the public or survey the impact of wastewater recycling should be accounted for in the logistical features of water recycling.

3.3.3 Public Confidence

There are several important factors pertaining to public confidence in a water recycling plan for Lake Mead.

- **Public Support:** In order to have a long-lasting water reclamation project, public support is needed to keep the project funded. The water reclamation plan must keep the public in mind through active communication and avoidance of dividing issues between the project and citizens.

3.4 The Wastewater Recycling Plan

With the complex of the issue in mind, our Wastewater recycling plan focuses on the following primary objectives for the states of Nevada, Arizona, and California:

1. Allocate resources for a preliminary survey of available land, possible applications for wastewater treatment in the region, and other important information vital for plan enactment.
2. Build and maintain treatment plants for irrigation, municipal plants, and environmental conservation using federal, local, and business resources.
3. Create new infrastructure to move wastewater for treatment and to move treated wastewater along to target locations.
4. Focus irrigation water recycling efforts on agricultural regions that are either most affected by the drought or are losing their traditional sources of water.
5. Focus municipal, commercial, and industrial water recycling efforts on densely populated regions such as urban centers to maximize gains made by treated wastewater.

6. Create a reserve of financial resources to either loan or offer to businesses as payment for water treatment facilities for business use.
7. Create a communications department to communicate with the community to register and resolve criticism.
8. Throughout the application of our water reclamation project, allocate resources to analyze the project's impacts for future study.

The fundamental concept of the wastewater recycling project is to conserve and use as much water as possible to alleviate the burden of Lake Mead in its outflow to the states of Arizona, Nevada, and California. The burden of Lake Mead must be alleviated due to Mead's lowering water elevations; independence must be established from Lake Mead both to prevent a water crisis and to generate the opportunity of augmenting the Lake's water levels artificially. In the first stage of the project, a preliminary survey of affected states would be conducted to acquire information on the region's relationship with water in areas like industry, agriculture, and the environment. Information like the availability of land for water reclamation facilities, the number of farms which use Lake Mead's water for irrigation, and the type of industries which use Lake Mead's water are crucial in determining the types of water quality to distribute from treated wastewater and in determining the logistical movement of treated wastewater. After the preliminary survey, enough information would be acquired for the main component of the water reclamation project built on three major uses: municipal, agricultural, and business-related. For each of these three major uses, the source of wastewater would coincide with one another with a large portion of wastewater being from domestic and industrial waste which will be transported to water treatment facilities designed for either standard or industrial treatment. The treated wastewater will be then moved to their required locations for either municipal, agricultural, or business-related pursuits. In anticipation of leftover treated wastewater, the unused water would be placed in the Lake Mead water reservoir after undergoing extensive water treatment similar to the one in industrial use. Overall, the plan seeks to increase inflow and decrease outflow to Lake Mead to alleviate Mead's rapidly declining water levels. The plan should at a minimum eliminate Lake Mead's outflows to the states, and the plan should hope to eliminate deficits in water levels during droughts with our models predictions serving as accurate predictors for future needs in water levels during predicted droughts.

3.4.1 Constructing Treatment Facilities

The primary method of treating wastewater is through the construction of large scale water treatment facilities through direct government involvement. For municipal and commercial wastewater, the primary wastewater treatment facility needed would be standard Sewage Treatment Plants (STPs). These treatment facilities remove contaminants from wastewater in a three-step process which completely cleans wastewater with a combination of physical, chemical, and biological treatment before discharging it for municipal use. On the other hand, the primary wastewater treatment facility required for industrial wastewater would be Effluent Treatment Plants (ETPs) which are used to clean industrial wastewater. They treat the unique form of wastewater effluent which flows out as a byproduct of industrial use. Their usefulness in sectors with high likelihoods of extensive chemical contamination such as the pharmaceutical industry or the textile industry make them the ideal candidate for industrial waste water treatment. Together, large scale STPs and ETPs should serve to facilitate a large majority of water needs in the affected areas. However, additional water treatment facilities may be needed to completely alleviate the stark decreases in water level for Lake Mead.

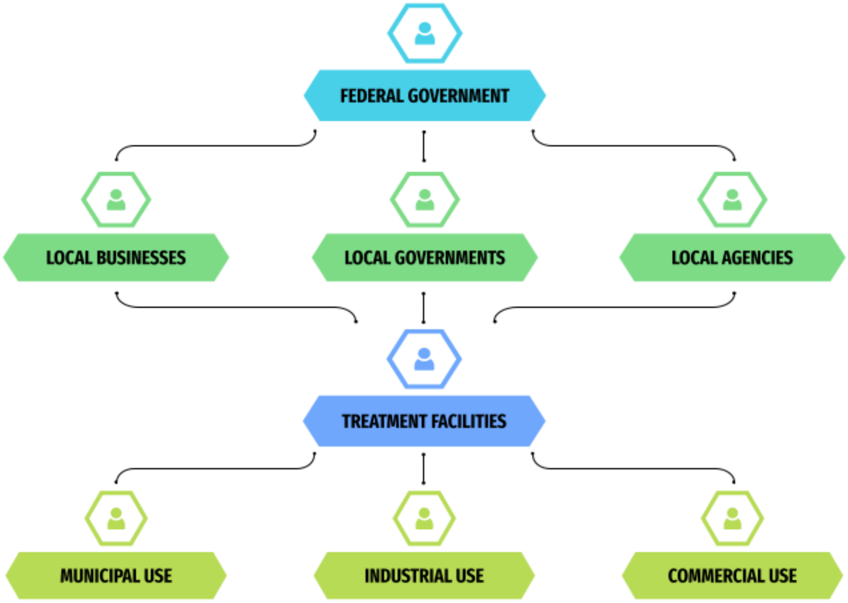

Figure 9: A decision tree highlighting the transfer of resources in our reclamation program.

3.4.2 Resource Management

In the event that government resources in managing the construction of water treatment facilities are shorthanded, the government could instead outsource the construction of wastewater treatment facilities and infrastructure to external companies specialized in the creation of water treatment facilities.

Financing could also facilitate the creation of small water treatment facilities such as Common and Combined Effluent Treatment Plants (CEPTs) which pools waste in a centralized plant for treatment. This decreases the costs of wastewater treatment by reducing the individual burden in maintaining a wastewater treatment facility. Outsourcing as a whole is important in our wastewater reclamation project as any lack of resources which are unavailable for the government could be sourced from local businesses or other water conservation organizations. Agencies like the United States Environmental Protection Agency and the American Water Works Association could lend expertise and local construction companies could lend field workers to create wastewater treatment facilities. Outsourcing also serves to give the water reclamation project flexibility in its processing as individual companies, industries, or farms could choose various types of water treatment facilities to be outsourced from the government for individual construction and adaptation, and outsourcing helps make the program compatible with current companies and government agencies by being inclusive of pre-existing resources and entities.

3.4.3 Communication

An important component of our water reclamation project is communication with the public and consumers. Active progress must be made towards connecting with the public to receive criticism and feedback on the project. If criticism is not taken into consideration during the project, the credibility and success of the project would deteriorate — irreparably damaging efforts in alleviating Lake Mead's water levels. Thus, funds must be allocated to communicating with the public to finetune the project's capabilities during the years of its application.

4 Plan Analysis

The overall impact of a project of this scale is determined by three major factors: the quality and quantity of the recycled wastewater, the breadth of available outsources to apply the recycled water, and the support of the recycled water program from customers and citizens alike. These factors can only be apparent after continued study of the wastewater recycling program. To address this issue, funds were allocated to the long term analysis of our program based on metrics from Arias et. al.'s [21] research on recycled water programs.

4.0.1 Quality and Quantity

The statistics metrics we used for determining the quality and quantity of recycled water were based on the U.S. Bureau of Reclamation statement that recycled water must meet the needs of the end user in terms of quality and quantity.

Water Quality: The US commonly uses three water quality parameters: total suspended solids (TSS), biochemical oxygen demand (BOD), and fecal coliform (FC). For each metric, the average annual measurement r is compared to the state standards. Water quality is deemed acceptable if the average annual measurements met or exceeded the state standard.

- **Test statement:** Annual average total suspended solid (TSS) concentrations for program meets or exceeds state standards for most restrictive use.

$$[TSS\,(mg/L)]_{program} \leq [TSS\,(mg/L)]_{state\,std}$$

- **Test statement:** Annual average biological oxygen demand/chemical biological oxygen demand meets or exceed state standards for most restrictive use.

$$[BOD\,or\,CBOD\,(mg/L)]_{program} \leq [BOD\,or\,CBOD\,(mg/L)]_{state\,std}$$

- **Test statement:** Annual average fecal coliform concentrations for program meets or exceed state standards for most restrictive use.

$$[FC\,(cfu/100ml)]_{program} \leq [FC\,(cfu/100ml)]_{state\ std}]$$

Recycled Water Portfolio Contribution: The metric measures the contribution recycled water makes to the overall water supply of affected regions.

$$\frac{Recycled\,Water\ consumed\ in\ a\ given\ year\ (ac-ft)}{Total\ water\ consumed\ in\ a\ given\ year\ (ac-ft)}$$

Recycled Water Growth Rate: The metric measures the change in the demand for recycled water by comparing the year over year volume of recycled water provided annually for a period of five years or more.

$$\frac{Volume\ of\ RW\ sold\ (current\ year,\ ac-ft)}{Volume\ of\ RW\ sold\ (previous\ year,\ ac-ft)}$$

4.0.2 Breadth of Application

There are several important statistical metrics pertaining to the variety of recycled water quality available in our water recycling plan.

Product Diversification: In meeting the specific needs of the public, recycled water should be tailored to match water treatment levels needed by the user' applications. The metric for product diversification is a binary classification for recycled water applications. If the application's water quality needs are met, the product is considered to be included in the program's capabilities to subsidize. Otherwise, the program is considered unable to meet the application's water quality needs. The product diversification metric would be the proportion of application's whose water quality needs are met.

Recycled Water Application Range: Based on the water demand for each application of recycled water, the metric measures the extent to which the program's water can be used for all possible recycled water applications.

$$\frac{Sum\ of\ all\ RW\ applications\ in\ service\ area}{Total\ number\ of\ possible\ RW\ applications\ in\ service\ area}$$

4.0.3 Customer and Public Support

The need for recycled water programs to understand the needs of customers and citizens is vital for program longevity. Metrics in determining customer and public support help suit the program's design to the needs of the public.

Customer Satisfaction: The metric measures the degree to which recycled water users are satisfied with the treated water from our program.

$$\frac{Number\ of\ customers\ satisfied}{Total\ number\ of\ costumers}$$

Customer Complaints: The metric measures the total number of customer complaints in a given year.

$$\frac{Number\ of\ customer\ complaints}{year}$$

Voter Support: The metric measures the level of public support in elections for the numerous factors involved in a water recycling project such as construction, financing, or final product.

$$\frac{Number\ \ of\ voters\ in\ support}{Total\ number\ of\ voters}(\%)$$

5 Conclusion

Before developing our plan, verified the methodology involved in the calculation of Lake Mead's relationships in water elevation, surface area, and volume. By determining water elevations using on field bathymetric data, surface area can be determined using an application of the Riemann integral, and volume relationships can be determined through the sum of consecutive frustums inherent in the lake's geometry. These confirmations led to our analysis of inflow, outflow, and loss factors for Lake Mead's hydrological cycle which was later condensed into a comprehensive equation for changes in water capacity for the reservoir.

In our model development, we used the previously created comprehensive equation to model Mead's future water levels using recently acquired water elevation data. Exploration of the available data revealed that recent water elevation patterns showed a tendency for high intensity droughts with low variability. Thus, we employed a sinusoidal model on the residuals formed by the linear regression model of a transformed representation of Lake Mead's monthly changes in water elevations. Subsequently, we applied our model to water elevation levels during both the recent drought period and water elevation data for the last fifteen years to predict future annual water levels in 2025, 2030, and 2050. Our models concluded that Lake Mead would face startling losses of water regardless of previous circumstance. In recognition of the need for water reclamation, we identified factors in the creation of a water recycling initiative in areas of treatment design, resource management, and public confidence.

We propose a federal, large-scale water reclamation project based on a two-pronged approach of direct federal construction of water treatment facilities through federal and local resources and outsourcing funds to pre-established agencies and local companies in the construction of practical water treatment technology. Finally, through long-term analysis using predetermined statistical measures, the impact of our water reclamation plan in treated water quality and quantity, breadth of application, and public support can be measured.

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